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EFFECT OF A PLANE ACOUSTIC PRESSURE WAVE ON A REINFORCED CYLINDRICAL SHELL

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An investigation of the strength of reinforced shells against pulsed loads is required to determine the limits of applicability of various designs in mechanical engineering and construction. This has resulted in a large number of publications on the development of theory and calculational method for ribbed shells (see review [1]). The effect of reinforcement ribs on the stress—strain state and the kinematic fields of cylindrical shells immersed in a fluid has been examined for transient excitation [2-4]. Special attention has been paid [2] to membrane stresses in the central cross section under the action of a plane wave. Radial displacements have been investigated [3] for axisymmetric loading in the center of the shell. Interaction of the fluid with the shell has been studied [4] according to the hypothesis of plane reflection. The behavior of flexure stresses on reinforced shells has hardly been studied.

Here we estimate the flexure and membrane stresses and the displacements of periodically reinforced shells during the transverse action of a plane translational pressure wave. A numerical solution of the problem is obtained by using a Fourier expansion in the angular coordinate and by using finite differences in the other coordinates. Numerical and analytical results are compared. The dynamic-response factor and the initial time at which these results coincide are determined.

1. The transient effect of a plane translational pressure wave is investigated for an infinitely long, thin, elastic cylindrical shell, which is periodically reinforced by ribs and immersed in an ideal elastic fluid. The shell is either empty or filled with the same fluid as surrounds it. The front of the incoming wave is parallel to the axis of the shell. The movement of the shell is described by linear equations of the Kirchhoff—Love theory; the excitations in the fluid are described by the wave equation for the velocity potential. The equations of motion for the m-th mode of oscillations along the angle θ have the form

$$\frac{1}{c^2}\frac{\partial^2 u_m}{\partial t^2} = \frac{\partial^2 u_m}{\partial x^2} - \frac{1-\nu}{2}\frac{m^2}{R^2}u_m + \frac{1+\nu}{2}\frac{m}{R}\frac{\partial v_m}{\partial x} + \frac{\nu}{R}\frac{\partial w_m}{\partial x},$$

$$\frac{1}{c^2}\frac{\partial^2 v_m}{\partial t^2} = \frac{1-\nu}{2}\frac{\partial^2 v_m}{\partial x^2} - \frac{1+\nu}{2}\frac{m}{R}\frac{\partial u_m}{\partial x} - \frac{m^2}{R^2}v_m - \frac{m}{R^2}w_m +$$

$$+ \frac{\delta^2}{12R^2}\left\{2(1-\nu)\frac{\partial^2 v_m}{\partial x^2} - \frac{m^2}{R^2}v_m - \frac{m^3}{R^2}w_m + (2-\nu)m\frac{\partial^2 w_m}{\partial x^2}\right\},$$

$$\frac{1}{c^2}\frac{\partial^2 w_m}{\partial t^2} = -\frac{\nu}{R}\frac{\partial u_m}{\partial x} - \frac{m}{R^2}v_m - \frac{w_m}{R^2} - \frac{\delta^2}{12}\left[\frac{\partial^4 w_m}{\partial x^4} - \frac{2m^2}{R^2}\frac{\partial^2 w_m}{\partial x^2} + \right.$$

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$$+\frac{m^4}{R^4}w_m-\frac{m}{R^2}(2-\nu)\frac{\partial^2 v_m}{\partial x^2}+\frac{m^3}{R^4}v_m\Big]-\frac{P_{\Sigma,m}}{\rho\delta c^2},$$

$$\frac{1}{c_0^2}\frac{\partial^2\varphi_m}{\partial t^2}=\frac{\partial^2\varphi_m}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi_m}{\partial r}-\frac{m^2}{r^2}\varphi_m+\frac{\partial^2\varphi_m}{\partial x^2}, \tag{1.1}$$

$$P_{\Sigma,m}=P_m-\rho_0\frac{\partial\varphi_m}{\partial t}\Big|_{r=R-0}+\rho_0\frac{\partial\varphi_m}{\partial t}\Big|_{r=R+0}.$$

Here t is time; u, v, and w are displacements along the x axis, the tangential direction θ, and the radial direction r; P is the pressure (assumed to be known) in the incoming wave; P_Σ is the total hydrodynamic pressure which acts on the shell from the inner and outer fluids; φ is the potential for the particle velocity of the fluid in the incoming, reflected, and diffracted waves; ρ_0 and c_0 are the density and sound speed in the fluid; ρ is the density of the shell material; R and δ are the shell radius and thickness; E is Young's modulus; ν is Poisson's ratio; and $c = \sqrt{E/\rho(1-\nu^2)}$ is the sound speed in a thin plate.

The ribs, which are located in sections $x = \pm L(2k+1)$ $(k = 0,1,\ldots)$, are modeled as rigid circular plates with a mass m_0. We assume that they only translate (without rotation) as rigid bodies. If U is the displacement $w = -U\cdot\cos\theta$ and $v = U\cdot\sin\theta$ are fulfilled in the section $x = \pm L\cdot(2k+1)$. This means that only the first harmonic in the Fourier expansion differs from zero:

$$w_m=v_m=0\ (m\neq 1),\ w_1=-U,\ v_1=U,\ U=(v_1-w_1)/2.$$

The equation for U is obtained from (1.1) by considering all forces which act on the side of the shell at the rib:

$$(1+\rho_*/\rho)/c^2\frac{\partial^2 U}{\partial t^2}=\frac{1-\nu}{4}\frac{\partial^2 v_1}{\partial x^2}-\frac{1-\nu}{4R}\frac{\partial u_1}{\partial x}-$$

$$-\nu\frac{\delta^2}{24R^2}\left(\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 w_1}{\partial x^2}\right)+\frac{\delta^2}{24}\frac{\partial^4 w_1}{\partial x^4}+\frac{P_{\Sigma,1}}{2\delta c^2\rho},\quad \rho_*=\frac{m_0}{4\pi R\delta L}.$$

In the sections $x = \pm 2Lk$ and $x = \pm L\cdot(2k+1)$, the following conditions are fulfilled:

$$\begin{gathered} x=\pm 2Lk\colon\ u_m=0,\ \partial\varphi_m/\partial x=0,\ \partial w_m/\partial x=0,\\ \partial v_m/\partial x=0,\ \partial^3 w_m/\partial x^3=0,\\ x=\pm L(2k+1)\colon\ u_m=0,\ \partial\varphi_m/\partial x=0,\ \partial^3 w_m/\partial x^3=0,\\ w_m=v_m=0\ (m\neq 1),\ w_1=-v_1=-U. \end{gathered} \tag{1.2}$$

Due to the symmetry, hereafter we will examine the behavior of the system in the interval $x\in[0, L]$. The following nonpenetration condition is fulfilled on the contact surface between the fluid and the shell:

$$r=R+0\colon\ \partial w_m/\partial t=\partial\varphi_m/\partial r+v_{rm};\ r=R-0\colon$$
$$\partial w_m/\partial t=\partial\varphi_m/\partial r,$$

where v_{rm} is the radial particle velocity in the incoming wave. The requirement that the potential be bounded leads to conditions on the axis r = 0

$$\frac{1}{c_0^2}\frac{\partial^2\varphi_0}{\partial t^2}=2\frac{\partial^2\varphi_0}{\partial r^2}+\frac{\partial^2\varphi_0}{\partial x^2},\quad \frac{\partial\varphi_0}{\partial r}=0,\quad \varphi_m=0\quad (m\geqslant 1). \tag{1.3}$$

Equations (1.3) have been obtained [5] with the Laplace transform in time and the Fourier transform in the axial coordinate, and also with a sequential limiting transition $(r\to 0)$. The initial conditions were homogeneous. In the incoming wave, which reaches the surface of the shell at time t = 0, the pressure is given in the form $P = P_0H_0(c_0t - R + r\cos\theta)$ where P_0 is the pressure amplitude at the wave front, and H_0 is the Heavyside function. Then the terms in the Fourier series for the pressure and the particle velocity in the fluid in the incoming wave are defined by the equations

$$v_{r,m}=\frac{P_0\varepsilon_m}{\pi\rho_0c_0}\left\{\frac{\sin(m+1)\theta_0}{m+1}+B_m\right\},$$

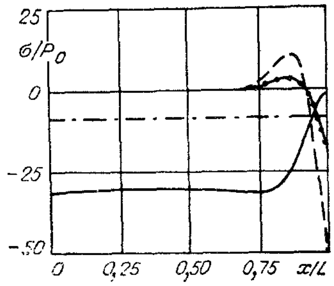

Fig. 1

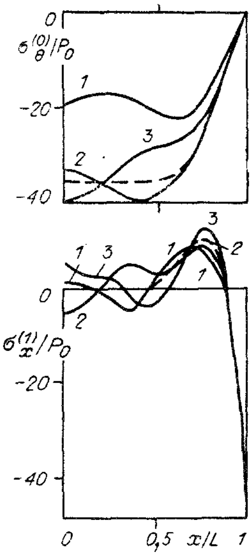

Fig. 2

$$B_0 = 0, \quad B_1 = \theta_0, \quad B_m = \frac{\sin(m-1)\theta_0}{m-1}, \quad \varepsilon_0 = 1, \quad \varepsilon_m = 2 \ (m \neq 0),$$

$$P_m = P_0 \frac{\theta_0}{\pi} \qquad (m = 0), \quad P_m = P_0 \frac{2}{\pi m} \sin m\theta_0 \qquad (m \neq 0),$$

$$\theta_0 = \begin{cases} \arccos(1 - c_0 t/R), & 0 \leqslant c_0 t/R \leqslant 2, \\ \pi, & c_0 t/R \geqslant 2. \end{cases}$$

2. We now derive several estimates of the perturbation parameters. Comparison of asymptotic ($t \to \infty$) and numerical solutions, derived [5] for smooth shells, showed that, due to the radiation of waves into the external fluid, the perturbation parameters for the zero harmonic (flexures, stresses, and total pressure) tend to their limiting values, which correspond to the static values. We assume that the presence of the ribs has a weak effect on P_Σ, and we determine P_Σ, from the problem of a plane wave acting on an infinite cylindrical without ribs [5]: $P_\Sigma = P_0/(1+\gamma)$. Here $\gamma = 2\rho_0 c_0^2 R/\rho c^2 \delta$ for a shell filled with fluid and $\gamma = 0$ for an empty shell.

By using the value of P_Σ for a smooth shell and throwing out inertial terms in (1.1), we obtain the static solution for the problem for a shell with ribs according to the zero mode which corresponds to the asymptote for $t \to \infty$:

$$\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x} + \frac{\nu}{R} w\right] = 0, \quad \frac{\nu}{R}\frac{\partial u}{\partial x} + \frac{w}{R^2} + \frac{\delta^2}{12}\frac{\partial^4 w}{\partial x^4} + \frac{P_\Sigma}{\rho c^2 \delta} = 0. \tag{2.1}$$

A solution [6] to the system (2.1) has been constructed under the assumption that $\partial u/\partial x + (\nu/R)\cdot w = 0$. We make an analogous argument without imposing this limitation. By using the boundary conditions (1.2) for $m = 0$, we solve (2.1) for a shell with a fluid:

$$\begin{gathered} w = -\frac{P_0}{\rho c^2 (1+\gamma)} \frac{R^2}{\delta} \frac{1}{1 - \nu^2 \chi/\beta} \{1 - A \cos\alpha x \operatorname{ch} \alpha x - B \sin \alpha x \operatorname{sh} \alpha x\}, \\ u = \frac{\nu P_0}{\rho c^2 (1+\gamma)} \frac{RL}{\delta} \frac{1}{\beta(1 - \nu^2\chi/\beta)} \left\{\frac{x}{L}\chi - \frac{A+B}{2} \sin \alpha x \operatorname{ch} \alpha x - \frac{A-B}{2} \cos \alpha x \operatorname{sh} \alpha x\right\}, \\ \beta = \frac{L\sqrt[4]{3(1-\nu^2)}}{\sqrt{R\delta}}, \quad \alpha = \frac{\beta}{L}, \quad \chi = \frac{\operatorname{ch} 2\beta - \cos 2\beta}{\operatorname{sh} 2\beta + \sin 2\beta}, \\ A = \frac{2(\cos\beta \operatorname{sh}\beta + \sin\beta \operatorname{ch}\beta)}{\sin 2\beta + \operatorname{sh} 2\beta}, \quad B = \frac{2(\sin\beta \operatorname{ch}\beta - \cos\beta \operatorname{sh}\beta)}{\sin 2\beta + \operatorname{sh} 2\beta}. \end{gathered} \tag{2.2}$$

Here the flexure [$\sigma_x^{(1)}$ and $\sigma_\theta^{(1)}$] and membrane [$\sigma_x^{(0)}$ and $\sigma_\theta^{(0)}$] stresses, which are determined by the formulas

$$\sigma_\theta^{(0)} = \rho c^2 \left\{\nu \frac{\partial u}{\partial x} + \frac{1}{R}(w + mv)\right\},$$

$$\sigma_x^{(0)} = \rho c^2 \left\{ \frac{\partial u}{\partial x} + \frac{\nu}{R}(w + mv) \right\},$$

$$\sigma_x^{(1)} = \rho c^2 \frac{\delta}{2} \left\{ \frac{\partial^2 w}{\partial x^2} - \frac{\nu m}{R^2}(mw + v) \right\},$$

$$\sigma_\theta^{(1)} = \rho c^2 \frac{\delta}{2} \left\{ \nu \frac{\partial^2 w}{\partial x^2} - \frac{m}{R^2}(mw + v) \right\},$$

for m = 0, we have the form

$$\begin{gathered}
\sigma_\theta^{(0)} = -\frac{P_0}{1+\gamma}\frac{R}{\delta}\{1 - A^* \cos\alpha x \operatorname{ch}\alpha x - B^* \sin\alpha x \operatorname{sh}\alpha x\}, \\
\sigma_x^{(1)} = \frac{P_0}{1+\gamma}\frac{R}{\delta}\sqrt{\frac{3}{1-\nu^2}}\{-A^* \sin\alpha x \operatorname{sh}\alpha x + B^* \cos\alpha x \operatorname{ch}\alpha x\}, \\
\sigma_\theta^{(1)} = \nu\sigma_x^{(1)}, \quad \sigma_x^{(0)} = -\nu\frac{P_0}{1+\gamma}\frac{R}{\delta}D, \\
A^* = A(1-\nu^2 D), \quad B^* = B(1-\nu^2 D), \quad D = \frac{1-\chi/\beta}{1-\nu^2\chi/\beta}.
\end{gathered} \tag{2.3}$$

For an empty shell we set $\gamma = 0$.

We note that for $\beta \geq \beta_0$ and $\beta - \alpha x \geq \beta_0$ the stresses (2.3) hardly depend on x, and Eqs. (2.2) and (2.3) are simplified significantly. Therefore in a rather wide range of variables x ($0 \leq x \leq x_*$) we can set

$$\begin{gathered}
\sigma_\theta^{(0)} = -\frac{P_0}{1+\gamma}\frac{R}{\delta}, \quad \sigma_x^{(0)} = -\nu\frac{P_0}{1+\gamma}\frac{R}{\delta}D, \quad \sigma_x^{(1)} = 0, \quad \sigma_\theta^{(1)} = 0, \\
w = -\frac{P_0}{\rho c^2(1+\gamma)}\frac{R^2}{\delta}\frac{\beta}{\beta-\nu^2}, \quad u = \nu\frac{P_0}{\rho c^2(1+\gamma)}\frac{R}{\delta}\frac{x}{\beta-\nu^2}.
\end{gathered} \tag{2.4}$$

The value of x_* is determined to the required numerical accuracy from the expression

$$\frac{L - x_*}{R} = \sqrt{\frac{\delta}{R}}\frac{\beta_0}{\sqrt[4]{3(1-\nu^2)}}.$$

For $\beta_0 = 3$ the error in Eqs. (2.4), compared to (2.3) and (2.2) is less than 1.5% for $x \leq x_*$.

Graphs of the functions (2.3) are shown in Fig. 1. The parameters of the problem are as follows: $\rho_0 = 0.128$, $c_0 = 0.3$, $\nu = 0.3$, $\rho = 1$, $c = 1$, $\rho_* = 0$, $L = 1$, $R = 1$, and $\delta = 0.01$. The solid curve corresponds to $\sigma_\theta^{(0)}$, the dashed to $\sigma_x^{(1)}$, the dash-dot to $\sigma_x^{(0)}$, and the curve with dots to $\sigma_x^{(1)}$. The flexure stresses $\sigma_x^{(1)}$ and $\sigma_\theta^{(1)}$ are essentially zero all the way to $x = x_*$; then comes a range of tension, and then a narrow region of very large compression near the ribs. The maximum value of $\sigma_x^{(1)}$ is attained at x = L:

$$\sigma_x^{(1)}\big|_{x=L} = -\frac{P_0}{1+\gamma}\frac{R}{\delta}\sqrt{\frac{3}{1-\nu^2}}.$$

The membrane stress $\sigma_\theta^{(0)}$ behaves differently. It is constant up to x_* and then changes smoothly to $\nu\sigma_x^{(0)}$. Its maximum (absolute) value is less than $\max|\sigma_x^{(1)}|$ by $\sqrt{3/1(1-\nu^2)}$ times and has the form

$$\sigma_\theta^{(0)}\big|_{x=0} = -\frac{P_0}{1+\gamma}\frac{R}{\delta}.$$

The stress $\sigma_x^{(0)}$ is constant along the shell and is less than $\max|\sigma_\theta^{(0)}|$ ($\sigma_x^{(0)} \approx \nu \cdot \max|\sigma_\theta^{(0)}|$). Thus, in a rib-reinforced shell the flexure stresses $\sigma_x^{(1)}$ are the most critical near the ribs. The maximum values of the stresses and the dimensions of the region of nonuniform stresses $L - x_*$ do not change as the distance between the ribs is increased. Conversely, the shell thickness has a substantial effect on the region of nonuniformity and on the maximum amplitudes of the stress.

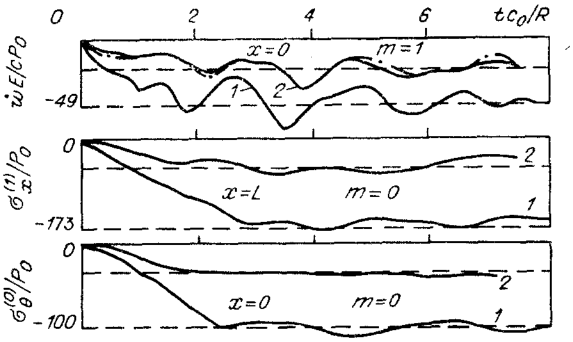

Fig. 3

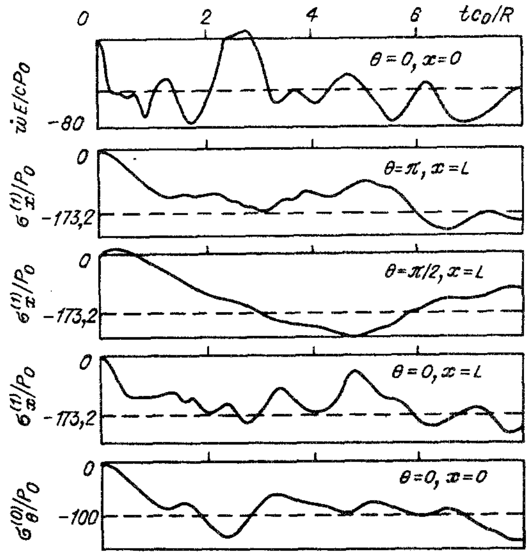

Fig. 4

Comparison of (2.2) and (2.3) with the results of [5] shows that the presence of ribs leads to a nonuniformity of the presses near them. Here flexure stresses arise, which are absent in the plane problem [5]; they are controlling in this problem, because they exceed the membrane stresses.

The asymptotic velocity $\dot{w}$ can be obtained from the following expressions. In the plane case at late times ($t \to \infty$) we have [5]

$$\dot{w}_1 \sim \frac{P_0}{c_0\rho_0\left(\frac{1}{2}(1+\varepsilon)+\gamma_1\right)}, \quad \gamma_1 = \frac{\rho\delta}{\rho_0 R}, \tag{2.5}$$

where $\varepsilon = 0$ for an empty shell and $\varepsilon = 1$ for a shell with fluid. The average mass per length of a shell with ribs is larger [than one without]. Therefore, instead of [2.5], we estimate the velocity by the quantity

$$\dot{w}_1 \sim \frac{P_0}{c_0\rho_0\left(\frac{1}{2}(1+\varepsilon)+\gamma_1+\gamma_2\right)}, \quad \gamma_2 = \frac{\rho_*\delta}{\rho_0 R}. \tag{2.6}$$

The limits of applicability of these asymptotes can be obtained by comparing them with a numerical solution of the problem, which makes it possible to investigate the transient strain process over the entire interaction of the wave with the shell.

3. The numerical solution to the problem was found by using an explicit four-way finite-difference grid. The parameters of the difference grid were chosen from the conditions for stability and for minimizing numerical divergence

$$c\tau = h_x \leqslant \frac{2R}{m}\sqrt{\frac{2}{1-\nu}} \qquad (m \neq 0),\ c\tau = h_x\ (m=0),$$
$$\frac{1}{c_0^2\tau^2} \geqslant \frac{1}{h_x^2} + \frac{1}{h_r^2}. \tag{3.1}$$

Comparison of time-dependent flexure stresses, calculated for $\delta = 0.01 \cdot R$ and various values of h_x, shows that the asymptotic values are practically exact, even for $h_x = R/40$. As h_x is increased, however, the error in calculating $\sigma_x^{(1)}$ increases, also on the critical side (maximum values of $\sigma_x^{(1)}$ are less than roughly 1.5 times the static values for $h_x = R/20$ and 2 times for $h_x = R/10$). Nonetheless, values of the displacements, velocities, and membrane stresses are completely satisfactory for a step $h_x = R/10$. The hydrodynamic forces can be determined with acceptable accuracy by increasing the step h_r compared to (3.1) by an order of magnitude.

Figure 2 shows the stress distributions $\sigma_\theta^{(0)}$ and $\sigma_x^{(1)}$ along the length of a fluid-filled shell at various moments in time. Curves 1-3 correspond to times $tc_0/R = 1$, 2, and 3. The dashed lines show the asymptote (2.3). The parameters of the problem are as follows: $\rho_0 = 0.128$, $c_0 = 0.3$, $\nu = 0.3$, $\rho = 1$, $c = 1$, $L = 1/2$, $R = 1$, $\delta = 0.01$, and $\rho_* = 0$. It can be seen that, starting with $tc_0/R = 2$, the numerical results oscillate relative to the asymptotic ones.

Figure 3 shows time-dependent curves of the flexure stresses $\sigma_x^{(1)}$ (at the point $x = L$) and the membrane stresses $\sigma_\theta^{(0)}$ (at the point $x = 0$) for the zero mode and the velocity $\dot{w}$ for the first mode, obtained for the same parameters as in Fig. 2. The dashed lines correspond to the stress asymptote (2.3) and to the velocity asymptote (2.6); the curves 1 correspond to the solution for an empty shell; and 2 to one filled with fluid; and the dot-dash curve corresponds to the problem with $\rho_* = 0.5$. The axisymmetric components of the flexure and membrane stresses and the first mode for the velocity reach asymptotic values (2.3) and (2.6) at the time $t = 3R/c_0$ and oscillate around them. The maximum amplitude of $\dot{w}$ for the first mode exceeds the asymptotic value by roughly 40%. Analysis of Eq. (2.6) shows that, for a thin shell and a rib mass comparable to the skin mass, the total mass is substantially less than the mass of the attached fluid. As a result, the asymptotic and numerical solutions for the velocity depend weakly on the rib mass. Actually, the dot-dash curves differ little from curve 2. The asymptotic values of the velocity for these parameters are 23.5 and 23.8, respectively.

Comparison of results calculated from the sum of the first three terms in the series ($m = 0, 1, 2$) and the sum of six modes ($m = 0, \ldots, 5$) showed that the amplitude of the flexure stresses do not differ by more than 7% in the rib region, where they are maximum. Thus, for acceptable accuracy in practical problems on the transverse action of a plane wave, it is sufficient to keep only three harmonics ($m = 0, 1, 2$). Figure 4 shows the process parameters for $m = 0, 1, 2$. Results of numerical calculations confirm the conclusion from Sec. 2 that the first mode of motion is the defining one for the velocity, while the maximum values for the membrane and flexure stresses are attained for the zero mode. The maximum amplitude of the flexure stresses is attained most rapidly for $\theta = \pi/2$ and $x = L$, and exceeds the analytical value (2.3) by 40%. Comparison of calculations for various values of L showed that changing the skin length hardly changes the maximum amplitudes of the flexure stresses.

Thus, the basic contribution to the stresses comes from the zero mode and the basic contribution to the radial velocity comes from the first mode. For $t \gtrsim 6R/c_0$, the asymptote describes the basic part of the excitations; for $t < 6R/c_0$ the increase in the excitations above the asymptote can be ~40%. In a reinforced shell, the axial flexure stresses are maximum in the rib region.

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